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Research Article

Understanding the Mathematical Problem Through Two Comprehension Strategies

İki Anlama Stratejisi Üzerinden Matematik Problemini Anlamak

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Abstract

Purpose: *This article discusses two strategies, underlining key information and identifying the givens and the unknowns, within a teaching module developed to improve students' problem comprehension skills. The aim of the study was to examine changes in students' problem-understanding behaviors following teaching sessions incorporating these strategies.* ***Method:*** *The main study was designed as a qualitative teaching experiment; however, this article presents a case study focusing specifically on these two strategies. Four different question forms were used to determine students' problem-comprehension behaviors before and after the teaching sessions.* ***Findings:*** *Prior to the intervention, students spent less time understanding the problem than solving it and demonstrated limited use of problem-comprehension strategies. Following the teaching sessions, an increase was observed in students' use of these strategies. A considerable number of students were able to understand the problems and formulate appropriate solution strategies. Students preferred the strategy of underlining key information over identifying the givens and the unknowns. Their reluctance to write appeared to influence their avoidance of the identifying the givens and the unknowns strategy.*

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Polya'nın problem çözme basamakları,
Problemi anlama,
Verilen ve isteneni belirleme,
Önemli bilgilerin altını çizma,
Ortaokul öğrencileri.

Öz

Amaç: *Bu makalede problemi anlamayı geliştirmeye yönelik geliştirilen bir öğretim modülünde yer alan iki strateji, önemli bilgilerin altını çizme ve verilen / isteneni belirleme, ele alınmıştır. Bu araştırmanın amacı, bu stratejileri içeren öğretim oturumları ardından öğrencilerin problemi anlamaya yönelik davranışlarındaki değişimi incelemektir.* ***Yöntem:*** *Nitel bir araştırma olarak kurgulanan ana araştırma öğretim deneyi deseni olarak planlanmıştır. Bu makale ise bu araştırmanın sadece iki stratejisine odaklanan bir durum çalışmasıdır. Araştırmada öğrencilerin öğretim oturumları öncesi ve sonrası problemi anlamaya yönelik davranışlarını belirlemek üzere dört ayrı soru formu kullanılmıştır.* ***Bulgular:*** *Öğretim oturumları öncesinde öğrenciler problemi anlamaya, problemi çözmekten daha az zaman ayırmakta, problemi anlama stratejilerini sınırlı düzeyde kullanmakta idi. Öğretim oturumları sonrasında ise öğrencilerin stratejileri kullanma düzeyinde artış olmuştur. Öğrencilerin önemli bir bölümünün problemi anladığı ve doğru çözüm stratejisini oluşturabildiği gözlenmiştir. Öğrenciler önemli bilgilerin altını çizme stratejisini, verilen ve istenenleri yazma stratejisine göre daha çok tercih etmişlerdir. Öğrencilerin verilen / istenen bilgileri yazma stratejisinden kaçınmalarında yazı yazmaya yönelik isteksizlikleri etkili olmuştur.*

Introduction

Considering the historical development of mathematics, mathematics emerged from individuals' efforts to solve problems encountered in daily life (Olkun & Toluk Uçar, 2006). According to the National Council of Teachers of Mathematics (NCTM) (2000), children's earliest experiences with mathematics begin through solving real-life problems. However, inappropriate instructional approaches, rote memorization, and repetitive practices may lead students to develop negative attitudes toward mathematics. As a result, mathematics may sometimes be perceived as a collection of abstract concepts unrelated to everyday life and therefore difficult or intimidating. Doing mathematics does not simply mean imitating procedures demonstrated by the teacher or solving numerous routine exercises (Van de Walle et al., 2014). Rather, mathematics education should incorporate students' diverse perspectives, ways of thinking, and creative approaches to problem-solving while preparing them for real life through mathematical experiences.

Howland (2001) states that problem solving is an essential component of mathematics curricula. Since one of the primary aims of mathematics education is to prepare students to solve real-life problems, problem solving occupies a central role in mathematics teaching. Similarly, Freudenthal (1968) argues that mathematics instruction should begin with real-life problems and continue through the problem-solving process. In the revised Secondary School Mathematics Curriculum introduced in 2024, problem solving is considered a high-level cognitive skill and is included among the five domain-specific competencies that the curriculum aims to develop (Ministry of National Education [MoNE], 2024). Within this curriculum, mathematical problem solving consists of four sub-skills: analyzing, interpreting, developing mathematical solutions, and

reflecting. Thus, the revised curriculum in Türkiye places particular emphasis on integrating problem-solving skills into mathematics instruction.

Problem-solving environments should enable students to conduct their own investigations and discuss possible solutions. To create such learning environments and manage the process effectively, teachers need to possess a strong understanding of the problem-solving process itself. Altun (2010) emphasizes that when children encounter a problem, they often attempt to recall a rule that can be directly applied to obtain the solution. However, although students may approach problem solving in this way, problem solving is not rule-based but systematic in nature. Initially, students should recognize this systematic structure under the teacher's guidance; over time, however, they should be able to manage the process independently.

Numerous frameworks and stages have been proposed to describe the problem-solving process. One of the most influential frameworks belongs to Polya (1945), whose problem-solving process consists of the following steps:

- Understand the problem
- Devise a plan
- Carry out the plan
- Look back

Similarly, Krulik and Rudnick (1993) divided the problem-solving process into five stages: reading and thinking, analyzing and planning, organizing a solution strategy, reaching a solution, and verifying the result. Bingham (1998) presented a more detailed classification of the problem-solving process, including recognizing the problem, gathering and organizing information, generating possible solutions, evaluating solution strategies, implementing the chosen strategy, and evaluating the process afterward.

Despite differences among these classifications, a common feature of all problem-solving frameworks is that the process begins with understanding the problem. Likewise, many researchers and educators identify understanding the problem as one of the most critical factors affecting successful problem solving (Cai, 2003; Jitendra et al., 2007; Karataş & Güven, 2004; Mayer, 1982; Polya, 1945; Stoyanova, 2005). Ulu (2011), for example, investigated the causes of elementary school students' errors while solving non-routine problems and found that the majority of errors stemmed from reading and comprehension difficulties (49.26%), followed by strategy selection errors (8.08%) and operational errors (2.62%). Similarly, Gökkurt and Soylu (2012) examined high school students' problem-comprehension levels during the problem-solving process and concluded that students were unable to use comprehension processes effectively. Although students could express the unknown using variables, they struggled to correctly identify the given information and explain the meanings of the values they obtained. In addition, students experienced difficulties identifying the information necessary for the solution and interpreting verbal statements in the problem.

These findings indicate that students at different grade levels may experience difficulties arising from reading comprehension during the problem-solving process. Existing research also demonstrates a strong relationship between reading comprehension and mathematical word-problem solving. Karakuş Aktan, Aslan, and Yalçın (2021), for instance, examined the effects of reading-comprehension instruction provided within Turkish language education on mathematical problem-solving skills. Conducted with 80 sixth-grade students from two classes in the same public school in Ankara, the study found that students in the experimental group who received reading-comprehension instruction outperformed those in the control group on a mathematics problem-solving achievement test.

Experimental studies conducted in Türkiye to improve problem-solving achievement (e.g., Altun et al., 2004; Altun & Arslan, 2006; Doğmaz Tunalı & Karabey, 2024; Durmaz & Altun, 2014; Gür & Hangül, 2015; Gürsan & Yazgan, 2020; Kılıç, 2009; Sulak, 2005; Okudur, 2023; Ulu et al., 2016; Yazgan & Bintaş, 2005; Yazgan, 2007) have primarily focused on strategy instruction. However, selecting an appropriate problem-solving strategy depends largely on understanding the problem itself. Understanding the problem is therefore a prerequisite for successful problem solving.

The Process of Understanding the Problem

The first step in problem solving is understanding the problem itself. Polya (1945, p. 18) explains the importance of understanding the problem before considering a solution strategy as follows:

"It is foolish to answer a question that you do not understand. … First of all, the verbal statement of the problem must be understood. … The student should also be able to point out the principal parts of the problem, the unknown, the data, the condition. … The student should consider the principal parts of the problem attentively, repeatedly, and from various sides. If there is a figure connected with the problem, he should draw a figure and point out on it the unknown and the data."

Constructing meaning from a text involves thinking about, inferring from, and evaluating the information presented. Processes such as examining, selecting, deciding, interpreting, analyzing, synthesizing, and evaluating are all essential components of comprehension. During this process, students should examine new information in light of their prior knowledge and mentally organize this information through active thinking. For this reason, instructional practices should prioritize activities that support text comprehension (MoNE, 2005). Similarly, Mih and Mih (2011) emphasize that comprehension plays an important role in making inferences.

One major obstacle students encounter in problem solving is difficulty understanding the verbal statements within mathematical problems (Tatar & Soylu, 2006). Kılıç (2009) found that fourth-grade students struggled to solve non-routine problems because they were unable to engage in meaningful reading. Consequently, they could not effectively carry out the later stages of the problem-solving process. Likewise,

studies conducted by Kroll and Miller (1993), Prakitipong and Nakamura (2006), and Tertemiz (1994) indicate that successful problem solvers generally possess higher comprehension skills, whereas there are no major differences between successful and unsuccessful students in terms of basic computational skills.

When students encounter a mathematical problem, they often read it quickly and immediately begin performing operations using the numbers provided in the text. However, the lengthy and demanding nature of the solution process may cause many students to give up on problem solving and develop negative attitudes toward it. Students who tend to seek rapid solutions are generally those who do not systematically follow problem-solving steps, approach problems superficially, and demonstrate lower levels of mathematical achievement (Aydoğdu & Ayaz, 2008). Encouraging students to spend more time understanding the problem may prevent them from wasting unnecessary time during the solution phase.

As the importance of problem solving has become increasingly recognized, greater attention has been devoted to examining the process of understanding the problem. Researchers have emphasized that comprehension is a process that can be improved through specific stages and strategies. According to Erümit (2014), understanding the problem involves identifying what is given and what is unknown, determining whether there is missing or unnecessary information, and identifying the types of information that can be inferred from the problem. Similarly, Erden (1984) reported that successful problem solvers employ strategies such as identifying the givens and the unknowns, restating the problem in their own words, drawing figures appropriate to the problem context, and representing the problem in different ways. Büyükalan Filiz and Abay (2017), in their study examining pre-service teachers' comprehension levels regarding non-routine problems, found that participants who could identify the givens and the unknowns, draw appropriate figures, and formulate a solution plan were more successful in solving the problems correctly.

Additional problem-comprehension strategies identified in the literature include rephrasing (Baykul, 2002; Erden, 1984), systematic list making (MoNE, 2009), visualization (Erden, 1984; Uesaka et al., 2007), constructing tables (Tertemiz & Çakmak, 2003), writing activities (Demircioğlu et al., 2010), simplifying the problem (MoNE, 2009), identifying missing or unnecessary information (Erümit, 2014), and underlining or marking key information (Fowler & Barker, 1974; Stephen & Brown, 2000).

Purpose of the Study

Understanding the problem, which constitutes the first and most fundamental stage of problem solving, is a prerequisite for progressing to subsequent stages. Both mathematics curricula and studies on problem solving emphasize the importance of problem comprehension. In the revised curriculum implemented in Türkiye, mathematical problem-solving skills include several subcomponents. Two of these subcomponents, analyzing and interpreting, include processes such as identifying relationships among

elements within the problem and expressing the situation in one's own words, both of which are closely related to understanding the problem (MoNE, 2024).

However, curriculum revisions alone are insufficient to identify and address the difficulties students experience during the understanding-the-problem stage, which is widely regarded as essential for successful problem solving. Research suggests that when students encounter a problem, they often attempt to move rapidly to the solution phase without sufficiently focusing on understanding the problem itself and consequently spend considerably more time on the solution stage (Aydoğdu & Ayaz, 2008). Students therefore require explicit instruction not only in problem solving but also in understanding the problem as an integral component of this process. Existing curricula and instructional practices appear to contain important gaps in this regard.

The broader study from which this article is derived focused on how students understand mathematical problems. A teaching module incorporating various comprehension strategies was developed to help students recognize the importance of understanding the problem and improve their comprehension processes. This article specifically focuses on two strategies included in that module: underlining key information and identifying the givens and the unknowns. The aim of the study was to compare students' use of these two strategies before and after the teaching sessions. In line with this aim, the study addressed the following research questions:

1. What actions did students take to understand problems prior to the teaching sessions, and how did they implement these strategies?

2. How did students' use of these strategies change after the teaching sessions?

Methodology

Research Design

This article forms part of a larger study designed to examine the effects of a teaching module intended to improve students' problem-comprehension skills. In order to investigate this process in depth, the study adopted a qualitative research approach. The primary research design was a teaching experiment, which is frequently used in mathematics education research. Teaching experiments focus on students' learning processes within instructional environments and aim to generate theories regarding both what students are capable of learning and how such learning can be facilitated. This design also seeks to explore students' mathematical thinking and identify ways to improve it (Steffe & Thompson, 2000).

In the broader study, the teaching experiment was designed to develop a teaching module that would improve students' levels of problem comprehension. In addition to the main study, the present article constitutes a case study focusing specifically on two comprehension strategies used during the teaching experiment: underlining key information and identifying the givens and the unknowns. Case studies involve collecting detailed and in-depth information about a specific real-life situation through multiple

sources of data, such as observations and interviews (Creswell, 2012). In this study, each of the two strategies was treated as a separate case.

Participants

The study was conducted in a rural school located in one of the outlying districts of Ankara. Convenience sampling, one of the purposive sampling techniques, was used to select the school. In convenience sampling, researchers select settings that are accessible and practical for the purposes of the study (Şimşek & Yıldırım, 2011). The researcher conducted the study at the school where she was employed as a mathematics teacher. This provided an opportunity to closely examine students' prior learning experiences, ongoing learning processes, and cognitive development within a teaching-experiment framework that integrated instruction and research.

The school served students transported daily from surrounding villages. Most students came from low socioeconomic backgrounds, with families primarily engaged in dairy farming and animal husbandry. Each grade level from fifth through eighth grade consisted of a single class containing approximately eight to twelve students. The participants in the study were seven seventh-grade students. Since there was only one seventh-grade class, all students in that class participated in the study.

Seventh-grade students were selected because they had already learned the four operations with natural numbers during the fifth and sixth grades yet continued to experience difficulties in understanding and solving word problems. It was determined that the curriculum objective "solves problems requiring operations with whole numbers" was appropriate for a teaching experiment focusing on problem comprehension. Two participants were female and five were male. Pseudonyms such as Gülçin, Selda, Göktan, Berke, Efe, Veli, and Aytaç were used instead of the students' real names.

Data Collection Tools and Process

Two tests were administered before the teaching experiment sessions in order to determine students' approaches to understanding mathematical problems. The first test aimed to identify how much time students allocated to understanding the problem and which comprehension strategies they used when they were free to choose their own approaches. The second test required students to use specific comprehension strategies, and their implementation of these strategies was subsequently analyzed. These tests were referred to as the Readiness 1 and Readiness 2 tests.

Following the administration of the readiness tests, the teaching sessions were conducted. The first three lessons focused on the importance of understanding the problem within the problem-solving process. Subsequently, four class hours were devoted to the strategy of underlining key information, and another four class hours focused on identifying the givens and the unknowns. After the teaching sessions, two separate worksheets containing questions related to the respective strategies were administered to the students. Contextualized real-life word problems were used in all

tests and worksheets. The questions used in the study are presented in the Findings section and Appendix.

Following the implementation of the worksheets, semi-structured interviews were conducted with the students in order to clarify and justify their responses. The researcher created a question pool for each strategy, developed the problems used during the teaching sessions, tests, and worksheets, and consulted experts to evaluate the appropriateness of the questions. Expert opinions were obtained from three academicians and eight secondary school mathematics teachers. The experts evaluated each question in terms of its suitability for the study's objectives and the students' grade level. Based on these evaluations, the final set of questions used in the teaching sessions and data collection tools was selected.

Ethical approval for the study was obtained from the Ethics Commission of Ankara University (Protocol Number 152-8). The research was conducted during the fall semester of the 2021–2022 academic year. Prior to the main implementation, a pilot study was conducted with a sixth-grade student using the research instruments. The pilot study was carried out to test the comprehensibility and suitability of the problems. Following the pilot study, the main research process was initiated.

Data Analysis

The data collected in this study were analyzed using both content and descriptive analysis. In descriptive analysis, research data are systematically coded and analyzed according to a pre-established conceptual framework and coding scheme (Strauss & Corbin, 1990). This study coded the problem-comprehension strategies employed by students while responding to questions on the Readiness 1 test, based on strategies identified in the literature. The coding encompassed strategies such as underlining key information, visualizing, and rephrasing.

Content analysis was used in the analysis of the other test and the worksheets. The purpose of content analysis is to make objective inferences from collected data through a systematic analytical process (Selçuk et al., 2014). Content analysis aims to identify patterns within the data and reveal underlying meanings and relationships (Altunışık et al., 2010). In this study as well, students' tendencies in using the strategies discussed in the article, as well as changes in these tendencies, were compared with the codes obtained from the content analysis. Codes or categories were developed from the organized data during the initial stage of the analysis.

The content analysis focused not only on the presence or absence of strategy use but also on the quality of students' implementation of the strategies. Particular attention was paid to how students interpreted the problem, which aspects of the problem they considered important, and whether their use of the strategies reflected a meaningful understanding of the problem situation. The analysis also aimed to identify changes in students' approaches to problem comprehension before and after the teaching experiment.

In some cases, the content of students' responses differed substantially before and after the teaching experiment. Consequently, in some cases, different coding structures and category systems emerged during the analysis of students' use of the strategies across these two phases. In the strategy of underlining key information, similar response patterns were observed across students' papers, allowing the markings to be categorized as "no underlining", "incomplete underlining" and "complete underlining". To illustrate this coding process, the category of "incomplete underlining" refers to cases in which students underlined only some, rather than all, of the key pieces of information in a problem. For instance, if a problem contained four key pieces of information, responses in which one, two, or three of these elements were underlined were coded separately and classified under this category. The analysis also noted which expressions were underlined (numbers or words).

In contrast, students' use of the strategy of identifying the givens and the unknowns differed considerably between the Readiness 2 test and the worksheets administered after the teaching experiment. Because students' worksheet responses contained more detailed and elaborated explanations, a different category system was required for the analysis of this strategy before and after the intervention. These coding and category systems were derived directly from students' responses and are explained in detail in the Findings section.

Interview data were analyzed together with students' written responses and were used to support the interpretation of the coding results. In particular, the interviews provided insight into students' reasoning, their perceptions of the strategies, and the factors influencing their decisions to use or avoid specific strategies. This triangulation enabled a more comprehensive understanding of students' problem-comprehension processes.

Researcher triangulation was employed during the analysis process. Approximately 26% of the data were independently coded by an additional researcher besides the primary researchers. Inter-rater reliability was calculated using the Miles and Huberman (1994) formula: Reliability = [Agreement/ (Agreement + Disagreement)] × 100. The agreement percentage was calculated as 96.30%. To enhance reliability and trustworthiness, direct quotations from interviews and excerpts from students' written responses were included throughout the study.

The Limitations of the Study

This study is qualitative research conducted with seven seventh-grade students enrolled in a rural middle school. Furthermore, the study focuses on only two comprehension strategies. Consequently, the results underscore the inclinations of these seven students regarding the two strategies.

Research Ethics

Ethical approval for the study was obtained from the Ethics Commission of Ankara University (Protocol Number 152-8). Since all participants were under the age of 18,

parental consent forms were prepared, and written consent was obtained from the students' parents prior to the study.

Findings

The findings are presented under two main headings: findings obtained before the teaching sessions and findings obtained after the teaching sessions.

Findings Before the Teaching Sessions

Two tests were administered prior to the teaching sessions in order to determine students' approaches to understanding mathematical problems. The findings obtained from these tests are presented below.

Students' time spent understanding the problem and the comprehension strategies they used

A three-question Readiness 1 test was administered to determine the amount of time students spent understanding the problems. The purpose of this test was to identify students' general approaches prior to the teaching sessions, specifically the time they allocated to understanding the problem and any comprehension strategies they already used. During the test, students were asked to indicate the moment when they finished reading the problem and began solving it. The researcher recorded these times during the implementation process, and the records were later verified using video recordings of the sessions. In this way, the time students devoted to understanding and solving the problems was determined. The findings were evaluated separately for each student.

The results for the three questions are presented for each student in Table 1. The average time students spent understanding and solving the problems is provided, together with the comprehension strategies they used while solving the questions.

Table 1: *Findings Related to the Readiness 1 Test*

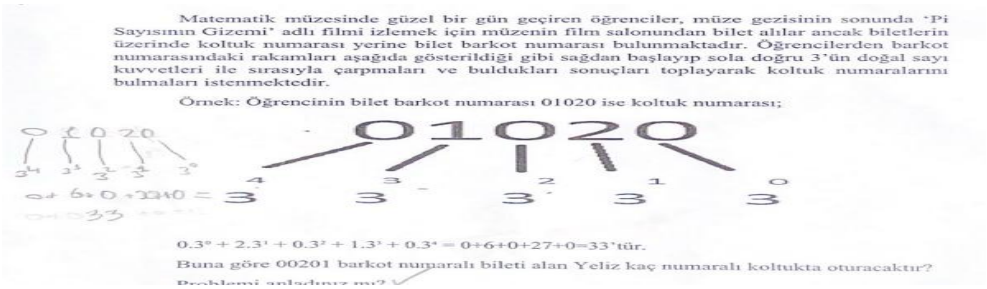
	Average Time Spent on Problem Understanding (min)	Average Time Spent on Problem Solving (min)	Number of Correctly Solved Questions (n=3)	Comprehension Strategies, if any
Gülçin	1.8	6.5	3	underlining key information, visualization
Selda	0.3	6.6	0	-
Göktan	2.8	1.8	0	-
Efe	1.9	7.3	2	underlining key information, identifying what is given
Aytaç	1.9	4.6	1	-
Veli	1.9	7	2	underlining key information
Berke	2.3	10.7	2	underlining key information, rephrasing, visualization

Except for one student (Selda), all participants generally spent more time solving the problems than understanding them. Göktan, on the other hand, made almost no attempt to solve the problems. In one problem, Veli failed to reach the correct result because of a computational error; however, in the problems students could not solve, they generally did not perform meaningful operations. Analysis of the students' written responses suggested that they often failed to understand the problems they could not solve.

The strategies of identifying what is given, underlining key information, visualization, and rephrasing appeared occasionally in students' written work; however, these strategies were generally implemented incompletely. Berke was the only student who actively used comprehension strategies in all three problems and attempted operations related to the solutions. Three students did not use any comprehension strategy in any of the problems. These findings suggest that students generally attempted to reach the solution quickly instead of allocating sufficient time to constructing meaning from the problem context.

The most frequently used strategy was underlining key information, while only a few students used visualization strategies. Although identifying the given information appeared in two responses (one by Berke and one by Efe), no paper included explicit identification of the unknown information. In Efe's response, for example, only the numerical expressions provided in the problem were rewritten, whereas the unknown was not represented in any way, and the problem could not be solved correctly.

Figure 1: *Efe's paper*


Matematik müzesinde güzel bir gün geçiren öğrenciler, müze gezisinin sonunda 'Pi Sayısının Gizemi' adlı filmi izlemek için müzenin film salonundan bilet alılar ancak biletlerin üzerinde koltuk numarası yerine bilet barkot numarası bulunmaktadır. Öğrencilerden barkot numarasındaki rakamları aşağıda gösterildiği gibi sağdan başlayıp sola doğru 3'ün doğal sayı kuvvetleri ile sırasıyla çarpmaları ve buldukları sonuçları toplayarak koltuk numaralarını bulmaları istenmektedir.

Örnek: Öğrencinin bilet barkot numarası 01020 ise koltuk numarası;

01020

4 3 2 1 0

3 3 3 3 3

0.3⁴ + 2.3³ + 0.3² + 1.3¹ + 0.3⁰ = 0+6+0+27+0=33'tür.

Buna göre 00201 barkot numaralı bileti alan Yeliz kaç numaralı koltukta oturacaktır?

Göktan and Aytaç experienced reading-comprehension difficulties not only in mathematics but also in other courses. When presented with lengthy contextualized word problems, they avoided using comprehension strategies and showed little evidence of understanding the problems in their written solutions. This finding suggests that reading-comprehension difficulties may directly limit students' willingness to engage with problem-comprehension strategies.

Students' use of the assigned strategies

The Readiness 2 test consisted of four questions, each accompanied by instructions specifying which comprehension strategy students should use. Two strategies commonly encountered throughout students' educational experiences—underlining key

information and identifying the givens and the unknowns—were included in this test. The findings are presented question by question with a focus on these two strategies.

In the first two questions, students were asked to underline key information. Content analysis resulted in the categories "no underlining," "incomplete underlining," and "complete underlining." No cases of unnecessary or excessive underlining were identified. The distribution of students' responses across these categories is presented in Table 2.

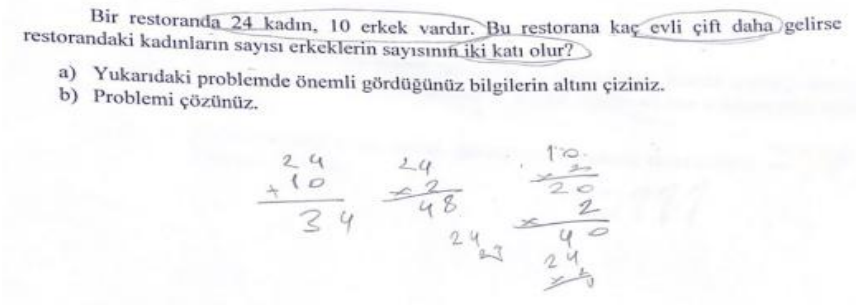
Table 2. *Distribution of Student Responses to Questions 1 and 2*

	No underlining	Incomplete underlining	Complete underlining
Question 1	-	Selda Göktan Veli	Aytaç Efe Berke Gülçin
Question 2	Veli	Göktan Selda Gülçin Efe Aytaç	Berke

Table 2 indicates that, even without explicit instruction, students were generally able to apply the strategy when prompted, although often incompletely. This may be explained by their previous exposure to underlining strategies during their school experience. In both questions, students primarily underlined numerical information.

In Question 1, many students whose responses were categorized as "incomplete underlining" failed to underline expressions such as "married couple" and "the number of women is twice the number of men," despite underlining all numerical values. This suggests that students recognized the importance of numbers but were less aware of the importance of relationships expressed verbally within the problem. In Selda's response, for example, she underlined the expressions "married couple" and "twice" but failed to underline the full verbal statement describing the relationship. Analysis of her solution indicated that she focused solely on the keyword "twice," mechanically multiplied by two, and approached the problem superficially. Selda's answer is shown in Figure 2.

Figure 2: *Question 1 Selda's paper*


Bir restoranda 24 kadın, 10 erkek vardır. Bu restorana kaç evli çift daha gelirse restorandaki kadınların sayısı erkeklerin sayısının iki katı olur?

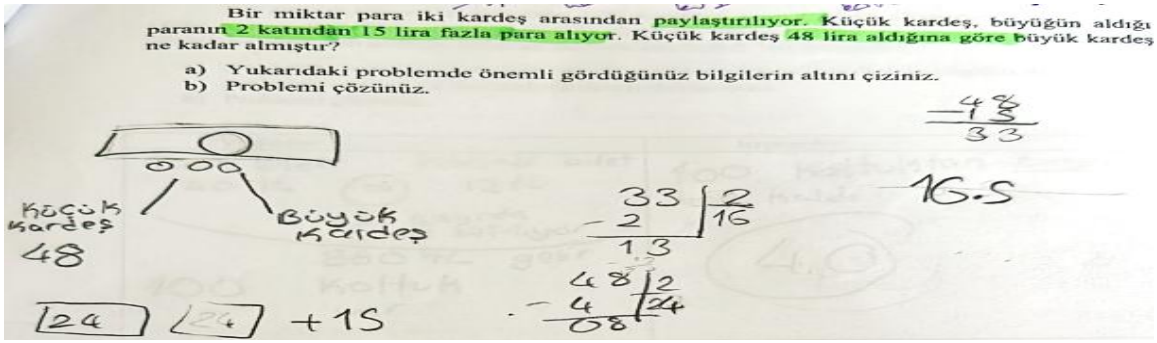
a) Yukarıdaki problemde önemli gördüğünüz bilgilerin altını çiziniz.
b) Problemi çözünüz.

Note. She underlined "24 women and 10 men", "more married couples", "will be twice"

A similar pattern was observed in Question 2. Students generally underlined numerical values while neglecting important verbal information. Students whose responses were categorized as "incomplete underlining" often failed to underline the statement "The younger brother receives 15 liras more than twice the amount received by the older brother." Only a small number of students fully underlined the key information. Most students underlined only the phrase "15 liras more than double" rather than the complete statement. The information specifying which sibling received more money played a critical role in solving the problem correctly. In this question, keywords appeared to mislead students during the solution process. This finding indicates that students relied heavily on isolated keywords during the solution process rather than interpreting the meaning of the entire statement within the problem context.

Gülçin was the only student who reached the correct solution despite being categorized under "incomplete underlining." Although she did not underline which sibling had more money, her written work indicated that she understood the problem correctly.

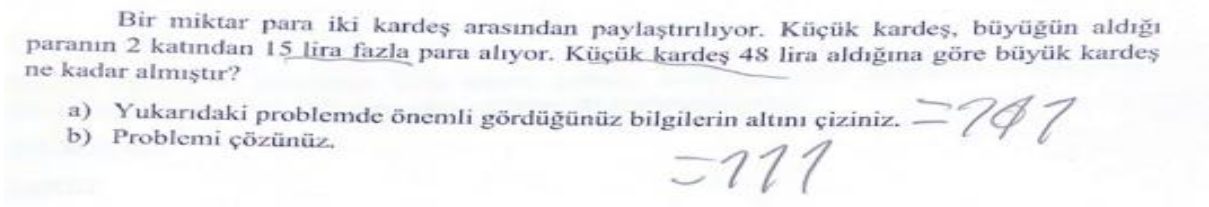
Figure 3: *Question 2 Gülçin's paper*



Note. She underlined "is divided", "15 liras more than twice", "received 48 liras"

Göktan's paper illustrates a different tendency. He underlined only numerical information and focused entirely on the given numbers during the solution process. As a result, he incorrectly multiplied 48 by 2 and added 15. Göktan's response demonstrates how focusing exclusively on numerical information without sufficiently processing verbal relationships can lead students to apply operations mechanically.

Figure 4: *Question 2 Göktan's paper*



Note. He underlined "15 liras more", "younger brother", "48 liras"

In Questions 3 and 4, students were asked to identify the givens and the unknowns before solving the problems. Analysis of students' responses produced the categories "givens not written," "givens written incompletely," "givens written completely," "unknown not written or written incorrectly," and "unknown written." The correctness of

the givens and the unknowns was evaluated separately while constructing these categories. The distribution of students' responses is presented in Table 3.

Table 3: *Distribution of Student Responses to Questions 3 and 4*

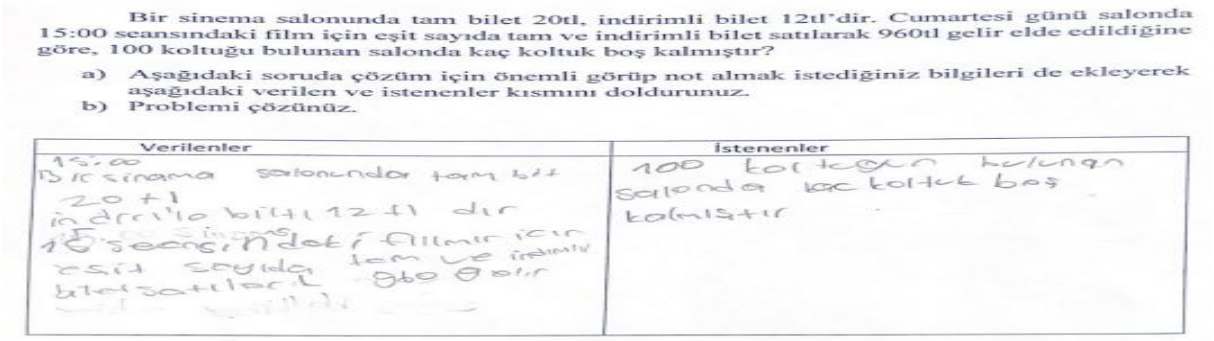
	Givens not written	Givens written incompletely	Givens written completely	Unknown not written or written incorrectly	Unknown written
Question 3	Göktan	Veli Aytaç Efe Berke Selda	Gülçin	Veli Göktan	Aytaç Efe Berke Selda Gülçin
Question 4	Veli Göktan	Aytaç Berke Gülçin Selda	Efe	Veli Göktan Selda	Aytaç Efe Berke Gülçin

Although students were explicitly instructed to identify the givens and the unknowns, Göktan resisted using the strategy and did not write either the givens or the unknowns in both questions. Similarly, Veli failed to identify the unknown information in either problem. Selda wrote the givens incompletely and identified the unknown incorrectly in Question 4.

In general, students attempted to identify the givens but often did so incompletely. Most responses were categorized under "givens written incompletely." Students typically wrote only numerical information in the "given" sections. Except for Aytaç and Gülçin, students failed to include the crucial information that "an equal number of discounted and full tickets were sold" in Question 3. One possible reason for this omission is that the statement did not contain explicit numerical information. The omission of non-numerical but conceptually important information suggests that students may not fully recognize the role of verbal relationships in structuring mathematical meaning.

Aytaç's response illustrates another tendency. In addition to identifying the important information regarding equal numbers of tickets, he also included irrelevant numerical information such as the starting time of the movie session. This may indicate students' tendency to perceive all numerical information in the problem as equally important. Regarding the unknowns, students generally copied the question sentence directly rather than reformulating it meaningfully.

Figure 5: *Question 3 Aytaç's paper*



Bir sinema salonunda tam bilet 20tl, indirimli bilet 12tl'dir. Cumartesi günü salonda 15:00 seansındaki film için eşit sayıda tam ve indirimli bilet satılarak 960tl gelir elde edildiğine göre, 100 koltuğu bulunan salonda kaç koltuk boş kalmıştır?

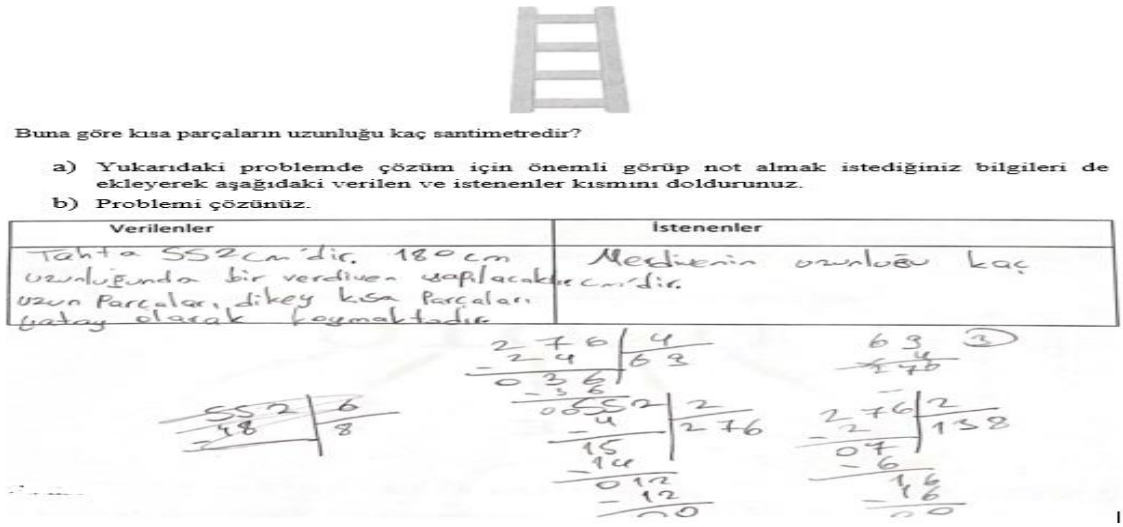
a) Aşağıdaki soruda çözüm için önemli görüp not almak istediğiniz bilgileri de ekleyerek aşağıdaki verilen ve istenenler kısmını doldurunuz.
b) Problemi çözünüz.

Verilenler	İstenenler
15:00 Bir sinema salonunda tam 20 indirimli bilet 12 dir 15 seans, eşit sayıda tam ve indirimli bilet satılarak 960 elde	100 koltuğun bulunan salonda kaç koltuk boş kalmıştır

Note. The given: 15:00 - In a movie theater, a full tick. is 20 TL - a discounted tick. is 12 TL - selling equal number of full and discount tickets for the movie in session 15 – 960 incomes, the unknown: how many seats were left empty in the theatre with 100 seats

A similar pattern appeared in Question 4. Most students were categorized under "givens written incompletely." Students generally wrote numerical expressions while omitting the verbal information related to the stair steps, which was essential for solving the problem correctly. Selda's response in Figure 6 demonstrates how misunderstanding the problem affected her identification of the unknown. Although the problem asked for the height of the steps, she identified the unknown as the length of the ladder. She also included incorrect information among the givens. Analysis of her solution showed that she did not understand the problem, and the incorrect givens and unknowns provided clear evidence of this misunderstanding.

Figure 6: *Question 4 Selda's paper*



Buna göre kısa parçaların uzunluğu kaç santimetredir?

a) Yukarıdaki problemde çözüm için önemli görüp not almak istediğiniz bilgileri de ekleyerek aşağıdaki verilen ve istenenler kısmını doldurunuz.
b) Problemi çözünüz.

Verilenler	İstenenler
Tahta 552cm'dir. 180 cm uzunluğunda bir merdiven yapılacak. Uzun Parçalar, dikey kısa Parçalar yatay olarak koymaktadır.	Merdivenin uzunluğu kaç cm'dir.

Note. The given: the board is 552 cm. A 180 cm long ladder will be made. put the long pieces vertically and the short pieces horizontally, the unknown: What is the length of the ladder?

Findings After the Teaching Sessions

Two worksheets were administered after the teaching sessions in order to examine changes in students' use of the two comprehension strategies. The findings obtained from these worksheets are presented below.

After the teaching sessions on underlining key information

The worksheet administered after the teaching sessions on underlining key information consisted of two questions. Since Efe was absent on the day of implementation, he did not complete the worksheet.

The coding structure used for analyzing this worksheet was similar to that used in the Readiness 2 test and consisted of three categories. During the analysis, the correctness of students' solutions was also taken into consideration. The distribution of students' responses according to the codes is presented in Table 4 together with whether the solutions were correct or incorrect.

Code 0: No underlining was made.

Code 1: Some information other than numerical data was underlined, but the key information was not identified completely; incomplete underlining was present.

Code 2: The key information in the problem was underlined completely.

Table 4. *Student Responses According to Codes*

	Code 0		Code 1		Code 2	
	Answer		Answer		Answer	
	True	*False*	*True*	*False*	*True*	*False*
Question 1	-	Veli	-	Göktan	Aytaç	Gülçin Selda Berke
Question 2	-	Selda	Veli	Göktan	Berke	Aytaç Gülçin

Unlike the findings from the Readiness 2 test, none of the students underlined only numerical information or marked irrelevant information in the worksheets administered after the teaching sessions. However, two responses, one for each question, were categorized under Code 0, indicating that no underlining strategy was used. In both responses, students failed to solve the problems correctly.

In Question 1, which asked students to determine the maximum possible number of three-person tickets sold for a concert with 70 guests, Veli explained during the interview why he had not underlined any information:

Rs: You didn't underline any information, why not?

Veli: I thought I could remember the problem because it was short.

The researcher asks the student to explain the problem.

Veli: It asks how many tickets for four-people and two-people will be sold until there are 70 people.

Rs: Could you read the last sentence aloud again?

Veli: "Since the number of guests is 70, how many tickets for three people have been sold at most?" Hmm… it says for three people. I always distributed two, three and four-person tickets equally.

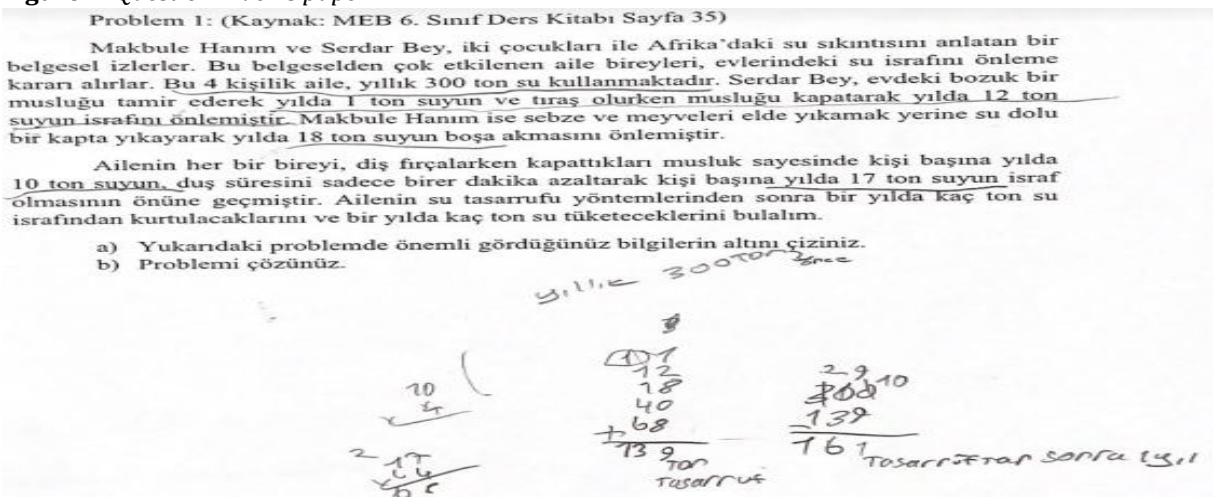
Rs: Is it possible not to buy any four-person tickets?

Veli: Yes.

As seen in the dialogue above, Veli stated that he did not underline the information because he believed he could retain it mentally. However, when asked to explain the problem, it became evident that he had not retained the information accurately. He overlooked the condition that at least one ticket from each category had to be sold and therefore incorrectly assumed that no four-person tickets needed to be purchased. When asked to reread the question, he recognized his mistake independently. Because he did not use any comprehension strategy, he moved rapidly from reading the problem to attempting a solution and consequently failed to understand the problem correctly.

In Question 2, Veli's response, which is seen in Figure 7, was categorized under Code 1. He underlined the expression "10 tons of water" but failed to underline the phrase "per person per year." Nevertheless, his calculations showed that he recognized the meaning of the expression and correctly multiplied by four to determine the amount of water saved for a four-person family. Although the unknown information itself was not underlined, his solution demonstrated awareness of the missing information, and he solved the problem correctly. This was the only response categorized under Code 0 or Code 1 that resulted in a correct solution.

Figure 7: *Question 2 Veli's paper*



Problem 1: (Kaynak: MEB 6. Sınıf Ders Kitabı Sayfa 35)

Makbule Hanım ve Serdar Bey, iki çocukları ile Afrika'daki su sıkıntısını anlatan bir belgesel izlerler. Bu belgeselden çok etkilenen aile bireyleri, evlerindeki su israfını önleme kararı alırlar. Bu 4 kişilik aile, yıllık 300 ton su kullanmaktadır. Serdar Bey, evdeki bozuk bir musluğu tamir ederek yılda 1 ton suyun ve tıraş olurken musluğu kapatarak yılda 12 ton suyun israfını önlemiştir. Makbule Hanım ise sebze ve meyveleri elde yıkamak yerine su dolu bir kapta yıkayarak yılda 18 ton suyun boşa akmasını önlemiştir.

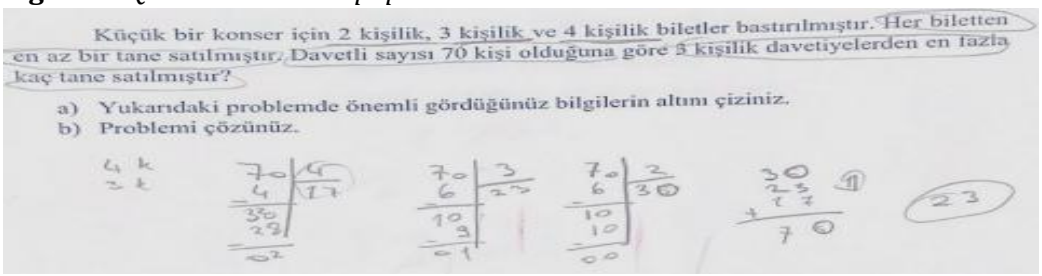
Ailenin her bir bireyi, diş fırçalarken kapattıkları musluk sayesinde kişi başına yılda 10 ton suyun, duş süresini sadece birer dakika azaltarak kişi başına yılda 17 ton suyun israf olmasının önüne geçmiştir. Ailenin su tasarrufu yöntemlerinden sonra bir yılda kaç ton su israfından kurtulacaklarını ve bir yılda kaç ton su tüketeceklerini bulalım.

a) Yukarıdaki problemde önemli gördüğünüz bilgilerin altını çiziniz.
b) Problemi çözünüz.

Note. He underlined "This 4-person family uses 300 tons of water annually.", "the waste of 1 ton of water per year", "12 tons of water per year by turning off the tap while shaving", "18 tons of water", "10 tons of water", "17 tons of water per year"

Most students' responses were categorized under Code 2, suggesting that the teaching sessions were effective in improving students' use of the strategy. However, there were also cases in which students fully implemented the strategy but still failed to reach the correct solution. Selda, for example, underlined all key information correctly in Question 1 but stated during the interview that she did not fully understand the problem.

Figure 8: *Question 1 Selda's paper*



Note. She underlined "for two, three and four people", "At least one of each ticket was sold", "number of guests was 70", "how many of the tickets for three people were sold at most"

For the remaining responses categorized under Code 2 but resulting in incorrect answers, interview data indicated that students generally understood the problems but made computational errors during the solution process. Berke, for instance, correctly understood the problem but made an error by adding the same quantity twice. Similarly, Gülçin understood both problems correctly but made procedural calculation errors. Such errors were considered acceptable within the scope of the study. Overall, the findings suggest that students who effectively applied the strategy generally understood the problems and were able to develop appropriate solution approaches, and compared with the readiness tests, students demonstrated a clearer awareness of the importance of verbal relationships and contextual statements after the teaching sessions.

After the teaching sessions on identifying the givens and the unknowns

The worksheet administered after the teaching sessions on identifying the givens and the unknowns consisted of two questions. In the analysis of this strategy, the givens and the unknowns were evaluated together, resulting in a four-category coding system. The distribution of students' responses according to these categories is presented in Table 5 together with whether the solutions were correct or incorrect.

Code 0: The student did not identify what was the givens and what was the unknowns in the problem or left this part blank

Code 1: The student wrote something about the givens and/or the unknowns but expressed the information incompletely or superficially

Code 2: The student correctly identified either the givens or the unknowns, but not both completely

Code 3: The student identified the givens and the unknowns in their own words

Table 5: *Student Responses According to Codes*

	Code 0		Code 1		Code 2		Code 3	
	Answer		Answer		Answer		Answer	
	True	*False*	*True*	*False*	*True*	*False*	*True*	*False*
Question 1	-	Selda Göktan	-	Aytaç Veli	-	Berke	Efe Gülçin	-
Question 2	-	Selda Göktan	Veli	Aytaç Berke	-	Efe	Gülçin	-

According to Table 5, Veli was the only student who reached the correct result among the responses categorized under Code 0 or Code 1. In general, students who did not apply the strategy or failed to internalize the given and the unknown information were unable to solve the problems correctly.

Selda and Göktan did not apply the strategy in either question, and their responses were categorized under Code 0. Göktan had also resisted using the strategy during the teaching sessions and readiness tests, and he demonstrated a similar tendency in the worksheets. During the interview, Göktan was asked to reread the problems, and it became evident that he did not fully understand certain parts of the text. However, with guidance and questioning from the researcher, he was eventually able to understand the problems. The interview revealed that Göktan avoided the strategy primarily because of his reluctance to write. Students' resistance to writing appeared to influence not only their use of the strategy but also the depth of their engagement with the problem itself.

Rs: What do you think about the strategy, were you able to use it?

Göktan: It was good, but I could not use it.

Rs: Do you have difficulty with writing?

Göktan: A little.

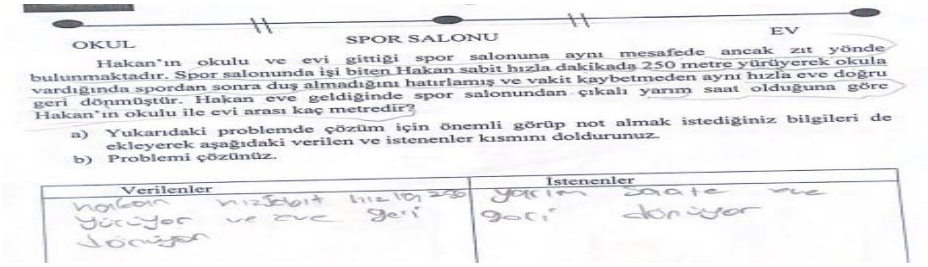
Rs: Is that why you do not want to write?

Göktan: Yes. (...) Underlining key information is easier.

Berke, whose responses were categorized under Code 1 and Code 2, also expressed concerns about writing during the interviews. He stated that he saw the strategy as a waste of time, he explained that it did not currently make sense to him and that he preferred underlining key information instead. Interestingly, Berke was one of the students who generally devoted time to understanding the problem, solved problems successfully, and used comprehension strategies even without prompting. Nevertheless, he still demonstrated reluctance toward writing-based strategies. This finding suggests that even students who are relatively successful problem solvers may avoid strategies they perceive as cognitively demanding or time-consuming.

Aytaç and Veli's responses in both questions were categorized under Code 1. Although they did not leave the given/unknown sections blank, their responses were superficial and did not demonstrate meaningful engagement with the strategy. For example, in Question 1, Aytaç wrote "He goes back home in half an hour" as the unknown, even though this statement represented the given information rather than the actual unknown. His tendency to use short and generalized expressions suggested that he did not devote sufficient attention to applying the strategy properly. It was observed that Aytaç wrote ''Hakan walks 250 at a constant speed and returns home'' in Question 1 in Figure 9 and ''He gave his monthly wages, and we did it by going that way'' in Question 2 as given information and used very short and general expressions in both questions and did not apply the strategy correctly.

Figure 9: *Question 1 Aytaç's paper*

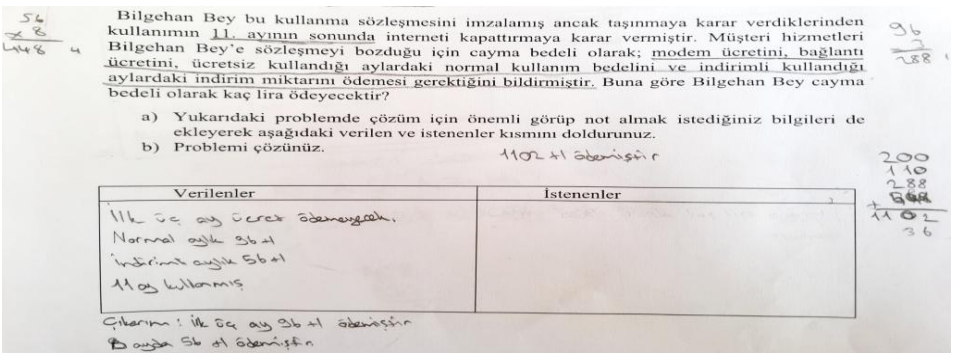


Although Aytaç displayed a positive attitude during the teaching sessions, interview data revealed that he answered the worksheet questions hastily and therefore did not carefully apply the strategy.

Responses categorized under Code 2 demonstrated partial internalization of the strategy. In these responses, students successfully identified either the givens or the unknowns in their own words but failed to identify both completely. Berke's response to Question 1 and Efe's response to Question 2 were categorized under Code 2. Partial use of the strategy appeared to support some degree of comprehension; however, incomplete internalization limited students' ability to construct a fully appropriate solution strategy.

In Efe's response, additional inferences were also observed. For example, he wrote statements such as "paid 96 TL for the first three months" and "paid 56 TL for eight months." However, he failed to infer the total discounted amount required to solve the problem correctly and therefore could not fully understand the problem. In addition, he did not identify the unknown information. Because the inferences he generated from the problem were insufficient, he could not construct the correct solution strategy. Although both students in this code performed operations on the data given in the problem, they were unable to fully implement the procedure required to reach the solution.

Figure 10: *Question 2 Efe's paper*



Note. The given: He will not pay for the first three months, normal monthly 96 TL, discounted monthly 56 TL, he used it for 11 months, inference: paid 96 TL for the first three months, paid 56 TL for eight months, The unknowns: -

All responses categorized under Code 3 resulted in correct solutions. When students internalized the strategy and meaningfully identified the givens and the unknowns, they were able to solve the problems successfully. During the teaching sessions, particular

emphasis had been placed on making inferences from the problems. Nevertheless, apart from Efe, no student explicitly wrote inferential statements on the worksheets.

Discussion and Conclusion

Although mathematics is often associated primarily with numbers and operations, improved reading-comprehension skills help students better understand word problems and naturally contribute to higher academic achievement (Helwig et al., 1999; Fuchs et al., 2000; Karakuş Aktan et al., 2021). Likewise, several studies examining factors affecting mathematics achievement have identified reading skills as an important component of success (Jiban & Deno, 2007; Lamb, 2010; Öztop & Toptaş, 2022; Saralar Aras & Demir, 2023). In this context, reading and understanding mathematical problems are essential components of problem-solving skills, which themselves constitute an integral part of mathematics curricula (Howland, 2001).

At the beginning of the present study, the amount of time students devoted to reading and understanding the problem was analyzed. The findings showed that students spent considerably more time solving problems than understanding them. The study also examined the comprehension strategies students employed before the teaching sessions. Although strategies such as underlining key information, identifying the givens and the unknowns, and rephrasing were occasionally observed, their use was generally limited and unsystematic. Despite the fact that these strategies are commonly encountered throughout students' school experiences, only one response included explicitly written givens, and none included explicitly written unknowns. Furthermore, students generally focused on numerical information while overlooking verbal relationships and contextual information that were essential for understanding the problems. These findings suggest that familiarity with a strategy does not necessarily translate into its meaningful application during problem solving. Rather than using these strategies as cognitive tools for organizing and interpreting information, the participating students appeared to employ them as surface-level procedural actions.

This interpretation is supported by Hegarty, Mayer, and Monk (1995), who reported that unsuccessful problem solvers tend to focus primarily on numerical values, whereas successful problem solvers pay greater attention to verbal relationships and contextual information. Similarly, the students in the present study concentrated heavily on numerical information during the problem-understanding stage while frequently overlooking relationships expressed through language. These findings suggest that mathematics instruction should provide students with more explicit opportunities to attend to verbal relationships, contextual meaning, and the structure of mathematical situations rather than focusing primarily on numerical information. The improvements observed following the teaching sessions further support the importance of explicitly developing students' problem-comprehension skills.

Following the teaching sessions, students demonstrated more effective use of both comprehension strategies. In particular, students began underlining not only numerical values but also important verbal expressions and relationships within the problems.

Students who fully implemented the underlining strategy were generally able to develop appropriate solution approaches. Those who failed to reach correct answers usually did so because of procedural or computational errors rather than misunderstanding the problems themselves. Considering that the participants had relatively low mathematics achievement levels and attended a disadvantaged rural school, the observed improvement in their approaches to problem comprehension is noteworthy. Although the findings cannot be generalized beyond the context of this study, they suggest that explicit instruction focusing on problem-comprehension strategies may support students in developing more meaningful approaches to mathematical problem solving.

This interpretation is consistent with previous research emphasizing the close relationship between reading comprehension and mathematical problem solving (Bayrak & Soylu, 2022; Erdem, 2016; Grimm, 2008; Gürsoy & Çeliköz, 2022; Karakuş Aktan et al., 2021; Öztürk et al., 2020; Tuohimaa et al., 2008). Reading comprehension has long been recognized as an important prerequisite for successful mathematical problem solving (Tatar & Soylu, 2006), and several researchers have argued that students' difficulties often arise from misunderstanding the problem rather than from performing mathematical operations (Serin & Korkmaz, 2018). The findings of the present study are consistent with this perspective. After the teaching sessions, students generally demonstrated improved understanding of the problems and were able to organize appropriate solution processes, even though some continued to make procedural or computational errors during the execution of the solution. Although the present study did not examine the direct effects of comprehension strategies on overall mathematics achievement, it provides evidence that understanding the problem and performing mathematical operations should be viewed as related but distinct components of problem solving. Improvement in one component does not necessarily eliminate difficulties in the other.

The findings regarding the strategy of identifying the givens and the unknowns were particularly important because they revealed students' resistance toward this strategy. In the readiness tests, students generally wrote only numerical expressions as givens and directly copied the question sentence as the unknown. Although identifying the givens and the unknowns is a strategy commonly introduced during elementary school, students' implementation of the strategy remained superficial before the teaching sessions. After the teaching sessions, some students began to internalize the information presented in the problems and restate the givens and the unknowns in their own words. These students also tended to solve the problems more successfully. Nevertheless, many students continued to avoid the strategy, and those who resisted its use generally experienced greater difficulty in understanding and solving the problems.

Previous studies have similarly emphasized that identifying the givens and the unknowns plays an important role in successful problem solving because it enables students to organize known and unknown information and establish meaningful mathematical relationships (Altun, 1995; Büyükalan Filiz & Abay, 2017; Kocaoğlu & Yenilmez, 2010; Van de Walle et al., 2014). While previous studies have reported that

students experience difficulties in identifying the givens and the unknowns (e.g., Kocaoğlu & Yenilmez, 2010), the findings of the present study provide a possible explanation for this difficulty. Interview data suggest that students' reluctance to engage in writing may be one factor underlying their resistance to using this strategy.

This finding can be interpreted in relation to the role of writing in mathematical problem solving. Unlike underlining key information, which allows students to mark information directly within the problem text, identifying the givens and the unknowns requires students to select relevant information, reorganize it, and express it in their own words. Consequently, the strategy places greater demands on both written production and cognitive processing. From this perspective, students' preference for underlining may not simply reflect a preference for one comprehension strategy over another; instead, it may indicate a tendency to choose strategies that require less written production and lower cognitive effort.

This interpretation is consistent with previous studies describing writing as both a cognitive and a metacognitive tool in mathematics education. Writing enables students to externalize their thinking, organize information, monitor their reasoning, and construct meaning during problem solving (Bangert-Drowns et al., 2004; Bell & Bell, 1985; Kuzle, 2013; Pugalee, 2004, 2005; Steele, 2007). Accordingly, the reluctance to write observed in the present study may have limited not only students' use of the identifying the givens and the unknowns strategy but also their opportunities to reorganize and reflect on the information presented in the problems. The findings therefore suggest that the differences observed between the two comprehension strategies may be explained not only by students' preferences but also by the different cognitive and written demands imposed by the strategies themselves.

These findings also have important implications for mathematics instruction. If students perceive writing-based comprehension strategies as unnecessarily demanding, they may avoid using strategies that actually promote deeper understanding of mathematical problems. Therefore, teachers should not assume that students will naturally adopt writing-based comprehension strategies once they are introduced. Instead, explicit instructional support should be provided to help students understand both how these strategies are used and why they contribute to meaningful problem comprehension. In addition, it would be beneficial for teacher education programs to include training on how writing-based comprehension activities can be integrated effectively into mathematics instruction.

The present study focused on Polya's understanding the problem stage and examined two comprehension strategies within a teaching module designed to improve students' problem-comprehension skills. Rather than investigating the effectiveness of the teaching module in terms of mathematics achievement, the study sought to explore how students approached these strategies before and after instruction. The findings showed that students' use of problem-comprehension strategies changed following the

teaching sessions, particularly with respect to their attention to verbal information and their implementation of the underlining strategy.

One of the important contributions of the study is the observation that students did not respond equally to different comprehension strategies. While most students readily adopted the strategy of underlining key information, many remained reluctant to identify the givens and the unknowns. The findings suggest that this difference may be associated not only with students' preferences but also with the different cognitive and written demands imposed by the two strategies. This interpretation highlights the importance of considering students' perceptions of strategy use, rather than focusing solely on whether a strategy is introduced during instruction.

These findings have several implications for mathematics education. First, problem comprehension should be addressed as an explicit instructional objective rather than being treated merely as the first step of problem solving. Students may benefit from systematic opportunities to learn how to interpret mathematical situations, recognize meaningful relationships, and organize information before attempting calculations. Second, because some comprehension strategies require students to externalize and reorganize their thinking through writing, teachers may need to provide additional support to encourage students to engage with these strategies meaningfully. In addition, curriculum materials and teacher education programs may benefit from placing greater emphasis on writing-based comprehension activities and their role in supporting mathematical reasoning.

The present study should also be interpreted in light of its scope. As a qualitative case study conducted with a small group of seventh-grade students in a single rural school, the findings are not intended to be generalized to broader student populations. Rather, they provide an in-depth understanding of how a particular group of students approached two problem-comprehension strategies within a specific instructional context. Future research may extend these findings by examining similar instructional processes with different age groups, educational settings, and larger samples, as well as by investigating how problem-comprehension strategies relate to students' mathematical achievement, reading comprehension, and metacognitive development.

Overall, the findings suggest that understanding a mathematical problem should not be viewed as a skill that develops automatically through repeated problem-solving practice. Instead, it appears to require explicit instructional support that helps students engage actively with the meaning, structure, and relationships embedded within mathematical problems. Supporting students in developing these comprehension processes may ultimately contribute to more meaningful and reflective mathematical thinking.

Ethical Approval

Ethical procedures were followed during research and publication process. Ethical approval was obtained at Ankara University with the protocol number of 152-8, dated May 17, 2021.

Author Contributions

Mehtap Çiçek: Conceptualization, Theoretical Background, Methodology, Data Collection, Data Analysis, Data Visualization, Writing

Ebru Aylar Çankaya: Conceptualization, Theoretical Background, Methodology, Writing, Review and Editing

Conflict of Interest

The authors declare no conflict of interest.

Plagiarism Checks

The article was reviewed using the relevant tools and met the BUEFAD similarity standards.

Artificial Intelligence Usage Statement

AI assistance was leveraged during the academic English editing stage of this study.

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Appendix

Readiness 1 Test

Question 1: Inform your teacher when you start reading and solving the problem!

Info:
Exponential Expression: A short representation of the repeated multiplication of a number by itself.

At the end of the museum visit, the students, who had a nice day at the Mathematics Museum, bought tickets from the movie hall of the museum to watch the movie 'The Mystery of Pi', but the tickets had a ticket barcode number on them instead of a seat number. The students are asked to multiply the numbers in the barcode number with the natural number powers of 3 from right to left as shown below and find their seat numbers by adding the results.

e.g.: The student's ticket barcode number is 01020 and the seat number is;



0.30 + 2.31 + 0.32 + 1.33 + 0.34 = 0+6+0+27+0=33
Accordingly, Yeliz who bought the ticket with barcode number 00201 will sit in which seat?

Question 2: Inform your teacher when you start reading and solving the problem!
A bacterial species multiply by dividing by two every 3 days. For example, at the end of the 3rd day, a bacterium divides by 2 and becomes 2 bacteria; at the end of the 6th day, these two bacteria divide in half and become 4 bacteria; at the end of the 9th day, these 4 bacteria divide in half and become 8 bacteria. According to this, at the end of which day 64 bacteria are formed from one bacterium in the laboratory?

Question 3: Inform your teacher when you start reading and solving the problem!
The price of the basketball that Kamil, Hayri and Şule want to buy is 12 TL more than Kamil's money, 18 TL more than Hayri's money and 5 TL less than Şule's money. According to this, how many TL differences are there between the money of the person with the most money and the person with the least money?

Readiness 2 Test

Question 1:
There are 24 women and 10 men in a restaurant. If how many more married couples come to this restaurant, the number of women in the restaurant will be twice the number of men?
a) Underline the information you consider important in the problem above.
b) Solve the problem.

Question 2:
An amount of money is divided between two brothers. The younger brother receives 15 liras more than twice as much as the older brother. Since the younger brother received 48 liras, how much money did the older brother receive?
a) Underline the information you consider important in the problem above.
b) Solve the problem.

Question 3:
In a movie theater, a full ticket is 20 TL, and a discounted ticket is 12 TL. Since equal numbers of full and discounted tickets were sold for the movie at the 15.00 session on Saturday and a revenue of 960 TL was obtained, how many seats were left empty in the theater with 100 seats?

a) Please fill in the given and unknown section below by adding the information that you consider important for the solution of the question.
b) Solve the problem.

Question 4:
Mehmet is going to build a 180 cm long ladder using a 552 cm long board. To do this, he divides the board into six parts, two equal parts are long and four equal parts is short, as shown in the picture below. Then he puts the long pieces vertically and the short pieces horizontally to make the ladder below. According to this, what is the length of the short pieces?

a) Please fill in the given and unknown section below by adding the information that you consider important for the solution of the question.
b) Solve the problem.

Underlining Key Information Worksheet

Question 1:
Tickets for two, three and four people were printed for a small concert. At least one of each ticket was sold. Since the number of guests was 70, how many of the tickets for three people were sold at most?

a) Underline the information you consider important in the problem above.
b) Solve the problem.

Question 2:
Makbule and Serdar watch a documentary about the water shortage in Africa with their two children. The family members were very impressed by this documentary and decided to prevent water waste in their homes. This 4-person family uses 300 tons of water annually. Serdar prevented the waste of 1 ton of water per year by fixing a broken tap in the house and 12 tons of water per year by turning off the tap while shaving. Makbule, on the other hand, prevented wasting 18 tons of water per year by washing vegetables and fruits in a container filled with water instead of washing them by hand.

Each member of the family prevented the waste of 10 tons of water per person per year by turning off the tap while brushing their teeth, and 17 tons of water per person per year by reducing the shower time by only one minute. Let's find out how many tons of water they will save in a year and how many tons of water they will consume in a year after the water saving methods of the family.

a) Underline the information you consider important in the problem above.
b) Solve the problem.

Identifying What is Given / Unknown Worksheet

Question 1:
Hakan's school and home are the same distance from the gym he goes to but in opposite directions. After finishing his work at the gym, Hakan walks 250 m per minute at a constant speed to school, remembers that he did not take a shower after the sport and walks back home at the same speed without wasting time. Since half an hour has passed since Hakan left the gym when he got home, how many meters are between Hakan's school and his home?

a) Please fill in the given and unknown section below by adding the information that you consider important for the solution of the question.
b) Solve the problem.

Question 2:

	24 Month Contractual Fee	Non-Contractual Fee
Modem Fee	-	200 TL
Connection Fee	-	110 TL
First Three Months/Every month	-	96 TL
Monthly Fee	56 TL	96 TL

An internet company is running a campaign for new users, whereby customers who sign a 24-month usage contract will pay nothing for the first three months, and for the remaining months, instead of paying the

normal monthly usage fee of 96 TL, they will pay a discounted monthly usage fee of 56 TL. The 200 TL modem fee and 110 TL connection fee will also be waived for customers who sign a usage contract.
Bilgehan signed this usage contract but decided to turn off the internet at the end of the 11th month as they decided to move. The customer service department informed Bilgehan that he would have to pay the modem fee, the connection fee, the normal usage fee for the free months and the discount amount for the discounted months. Accordingly, how many liras will Bilgehan pay as withdrawal fee?

a) Please fill in the given and unknown section below by adding the information that you consider important for the solution of the question.
b) Solve the problem.